Chapter one-sets

Representation of sets

1) Tabulation method

Examples:

1) A ={1,3,7,10}

2) The set of student in computer science

2) Rule method

Examples:

1) A = { x; x is student in university of Diyala}

2) B = {x ; x^2-3x-2= 0}

If a is an element in the set A then we write a ∈A. If a is not an element in the set A then we write a ∉ A.

Examples:

1) A = {1,3,4,5,6,7} , 4 ∈A, 2 ∉A.

2) B = {x ; x is even}. 2 ∈ B, 5 ∉ B

Finite and infinite set

Examples:

1) The set of days in week is finite

2) A= {2 , 4, 6 , 8 , ….} is infinite

3) B = { x ;x is river in the earth} is finite

Natural numbers N = {0,1,2,3,4,….}

Integer numbers Z = { …-3,-2,-1,0,1,2,3,4,….}

Positive numbers Z^+ = {1,2,3,4,….}

Negative numbers Z^{-} = {…-3,-2,-1}

Rational numbers Q = {$\frac{a}{b}$, a , b ∈ Z, b ≠0}

Real numbers

Complex numbers C = {x+iy , x , y ∈ R, i = $\sqrt{-1}$}

Even numbers Z_e = {x , x = 2n , n ∈Z} = {…. -4, -2, 0, 2 , 4 …}

Odd numbers Z_o = {x , x = 2n+1, n ∈Z}={…,-3, -1, 1, 3 , 5,… }

The set which contains no element is called empty set or null set and denoted by ∅

Examples:

1) The set of people in the world who are older than 200 years.

2) A = { x; x ∈ Z_o ; $x^2 = 4$}

3) B = { x; x ∈ N ; $2 < x < 3$}

4) C = { x; x ∈ Z_e ; $x^2 = 15$}

5) D = { x; x ∈ Z ; $0 < x < 1$}

The set A is a subset of a set B denoted by (A ⊆ B) if each element in A belong to B

Examples:

1) A = {1,3,5} is a subset of B = {1,2,3,4,5}

2) G = {x ;x is even}is a subset of F = {x ;x is a positive power of 2}

The null set is a subset of every set

If there is at least one element in A that is not a member of B then A is not subset of B, denoted by A ⊈ B.

Example:

A = { x; x ∈ Z ; $-3 < x < 3$} = { -2 , -1 , 0 , 1 , 2 } ⊆ Z , but A ⊈N.

Example: A ={2,3,4,5}, B = {2,4,5,6}, A ⊈ B.

If A ⊆ B and B ⊆ A, then A = B

Example: A = {x ; x^2-3x+2 = 0}, B = {1,2}, A = B

If A ⊆ B , A ≠ B, then A is a proper subset of B , denoted by A ⊂ B.

Example: N ⊂ Z ⊂ Q ⊂ R ⊂ C

Properties: Let A , B , C be sets then

1) A ⊆ A

2) If A ⊆ B, B ⊆ C then A ⊆ C

3) If A ⊂ B, B ⊂ C then A ⊂ C

If the set is a subset from big set or another set then this set is called universal set , denoted by U

Example: A = {1,3,5}, B = {2,4,6}, C ={2,9,10}

U_1 = {1,2,3,4,5,6,7,8,9,10} , U_2 = N , U_3 = Z

The set of all subset of any set S is called the power set of S, denoted by P(S) or 2^S

If S has n elements then P(S) has 2^n elements

Examples:

1) M = {a,b} , P(M) = {∅, {a}, {b} , M}

2) T = {4, 7, 8}, P(T) = {∅,{4},{7},{8},{4,7},{4,8},{7,8},T}

Properties: 1) If $A \subseteq B$, then $P(A) \subseteq P(B)$

2) If $P(A) \subseteq P(B$ then $A \subseteq B$

3) If A = B then P(A) = P(B)

4) If P(A) = P(B) then A = B

Operation on sets

1) The intersection of sets

Let A , B be sets, the intersection of A and B (denoted by $A \cap B$) is the set $A \cap B = \{ x ; x \in A \text{ and } x \in B\}$

Example : A= { 2 , 3} , B= { 3,5,7} , $A \cap B$ = { 3}

Properties: Let A , B be sets then

1) $A \cap B \subseteq A$, $A \cap B \subseteq B$

2) $A \cap A = A$

2) $A \cap B = B \cap A$

3) $(A \cap B) \cap C = A \cap (B \cap C)$

4) $A \cap \emptyset = \emptyset$

5) $A \cap U = A$

7) $P(A \cap B) = P(A) \cap P(B)$

Let A , B be sets , A and B is called disjoint if $A \cap B = \emptyset$

Examples: 1) Even numbers and odd numbers.

2) A = {1,3,4,5}, B = {2,6,7,9}

2) The union of sets

L et A , B be sets, the union of A and B (denoted by A ∪ B) is the set A ∪ B = { x ; x ∈A or x ∈B}

Properties: Let A , B be sets then

1) A ⊆ A ∪ B , B ⊆ A ∪ B

2) A ∪ A = A

3) A ∪ B = B ∪ A

4) (A ∪ B) ∪ C = A ∪ (B ∪ C)

5) A ∪ ∅ = A

6) A ∪ U = U

7) P(A) ∪ P(B) ⊆ P(A ∪ B)

Distribution laws

Let A , B , C be sets then

1) A ∩ (B ∪ C) = (A ∩ B) ∪(A ∩ C)

2) A ∪ (B ∩ C) = (A ∪ B) ∩(A ∪ C)

3) Difference of sets

Let A , B be sets, the difference of A and B (denoted by A - B) is the set A - B = { x ; x ∈ A and x ∉ B}

A - B ⊆ A, A - B ⊈ B

A-B ≠ B-A

Example: A = {3,47,8,9}, B = {1,2,3,4,6,10}

A-B = {7,8,9} , B-A = {1,2,6,10}

In case $B \subseteq A$, $A - B$ is called complement of B in A (denoted by B_A^C)

$B^C = B_U^C = U - B = \{ x ; x \in U \text{ and } x \notin B\}$

Properties: Let A , B be sets then

1) If $A \subseteq B$ then $B^C \subseteq A^C$

2) $(A^C)^C = A$

Demorgan laws

Let A , B , C be sets then

1) $(A \cup B)^C = A^C \cap B^C$

2) $(A \cap B)^C = A^C \cup B^C$

Properties: 1) $A \cup A^C = U$, $A \cap A^C = \emptyset$

2) $\emptyset_U^C = U$, $U_U^C = \emptyset$

Properties: Let A , B be sets then

1) $A - B = A - (A \cap B)$

2) $A - B = B^C - A^C$

3) $A - B = A \cap B^C$

4) The delta

$A \triangle B = (A \cup B) - (A \cap B)$

$A \triangle B = B \triangle A$

Chapter two-logic

Statements

1) Baghdad is the capital of Iraq (T) Statement

2) 2+1 = 4 (F) Statement

3) π is the rational number (T) Statement

4) Where do you live not statement

Compound Statements

Let p, q b two simple Statements. If we take p and q together, we get new Statement called Compound Statements.

There are three connectives

1) And $\wedge$

2) Or $\vee$

3) Not $\sim$

The truth tabular of p∧q

p	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

The truth tabular of pVq

p	q	pVq
T	T	T
T	F	T
F	T	T
F	F	F

The truth tabular of ~p

p	~p
T	T
F	T

Example: Write the truth tabular of 1) p∧~q   2) ~pV~q

p	q	~q	p∧~q
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

p	q	~p	~q	~pV~q
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Conditional statements

The statements if p then q is called if statements and denoted by p → q

The truth tabular of p → q

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

Example: Write the truth tabular of 1) p →~q
2) (pVq) →(pΛq)   3) P→(q→r)

p	q	~q	p→~q
T	T	F	F
T	F	T	T
F	T	F	T
F	F	T	T

p	q	pVq	pΛq	(pVq) →(pΛq)
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

p	q	r	q→r	P→(q→r)
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	T
F	F	T	T	T
F	F	F	T	T

The Biconditional statements

$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

The truth tabular of $p \leftrightarrow q$

p	q	$p \to q$	$q \to p$	$p \leftrightarrow q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Example: Write the truth tabular of 1) $(p \wedge q) \leftrightarrow (p \vee q)$ 2) $\sim (p \wedge q) \vee \sim (q \leftrightarrow p)$

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \to (p \vee q)$	$(p \vee q) \to (p \wedge q)$	$(p \wedge q) \leftrightarrow (p \vee q)$
T	T	T	T	T	T	T
T	F	F	T	T	F	F
F	T	F	T	T	F	F
F	F	F	F	T	T	T

p	q	$p \wedge q$	$\sim (p \wedge q)$	$q \leftrightarrow p$	$\sim (q \leftrightarrow p)$	$\sim (p \wedge q) \vee \sim (q \leftrightarrow p)$
T	T	T	F	T	F	F
T	F	F	T	F	T	T
F	T	F	T	F	T	T
F	F	F	F	T	F	T

The Conditional statements is called Tautology if it is always true

Example: $p \rightarrow (p \vee q)$

p	q	pVq	p→(pVq)
T	T	T	T
T	F	T	t
F	T	T	t
F	F	F	T

The Conditional statements is called Contraduction if it is always false

Example: $(p \rightarrow q) \wedge (p \wedge \sim q)$

p	q	~q	p → q	p∧~q	(p → q) ∧ (p∧~q)
T	T	F	T	F	F
T	F	T	F	T	F
F	T	F	T	F	F
F	F	T	T	F	F

Chapter three-Relation

Cartesian product

Def: Let A, B be two sets, the cartesian product of A and B is the set A×B = {(a,b), a ∈ A , b ∈ B}

Example: Let A = { 1 , 2 , 3}, B = { a , b}

A×B = { (1 , a) , (1 , b), (2 , a), (2 , b) , (3 , a), (3, b)}

B×A = { (a , 1) , (a , 2), (a , 3), (b , 1) , (b , 2), (b, 3)}

Remarks

1) Let A be set contains n elements, B be set contains m elements, then A×B contains m× n elements

2) A×B≠ B×A

Proposition Let A , B , C and D be sets , then

1) A ×(B∪C) = (A×B) ∪(A×C)

2) A ×(B∩C) = (A×B) ∩ (A×C)

3) A ×(B∪C) = (A×B) ∪(A×C)

4) A ×(B−C) = (A×B)−(A×C)

Def: Let A, B be two sets, each subsets of A×B is called relation from A to B denoted by R(R ⊆ A×B)

The relation is called relation on A, If R ⊆ A×A.

Example: Let A = { 1 , 2 , 3}, B = { a , b}

R = {(1, a) , (1 , b) , (3 , a)} is relation since R ⊆ A×B

R_1 = {(1, b) , (3 , a) , (b , 2) , (2, a)} is not relation since R ⊄A×B

Example: Let W = { a , b , c},

R = {(a, b) , (a , c) , (c , c) , (c , b)} is relation on A since R ⊆ W×W.

Remark: Let A be set contains n elements, B be set contains m elements, then there is 2^{nm} different relation from A to B

Def: Let A = { a_1 , a_2 , a_3 , a_4 , …} , B = { b_1 , b_2 , b_3 , b_4 , …}

R = { (a_1 , b_1) , (a_2 , b_2), (a_3 , b_3),(a_4 , b_4) , (a_5 , b_5), (a_6, b_6) , …}

The domain of the relation is the first element

The range of the relation is the first element

Domain R ⊆ A, Range R ⊆B

Example: Let A = { 1 , 2 , 3}, B = { 0 , 3 , 4 , 5 , 6}

R_1 = { (1 , 0) , (2 , 5) , (3 , 6)}

Dom(R_1) = { 1 , 2 , 3} = A

Range(R_1) = { 0 , 5 , 6} ⊆ B

R_2 = { (1 , 3) , (1 , 5) , (3 , 4) , (3 , 0)}

Dom(R_1) = { 1 , 3} ⊂ A

Range(R_1) = { 3 , 5 , 4, 0} ⊂ B

A relation R on set A is called

1) Reflexive if foe each a ∈ A , (a , a) ∈ R

2) Symmetric if (a , b) ∈ R then (b , a) ∈ R

3) Transitive if (a , b) ∈ R ∧ (b , c) ∈ R then (a , c) ∈ R

Example: Let A = { 0, 1 , 2 , 3},

R_1 = { (0,0), (1,1) , (2,2) , (3,3), (1 , 3) , (3,1) , (0 , 2) , (2 , 0)} reflexive , symmetric , transitive

R_2 = { (0,0), (1,1) , (2,2) , (3,3), (1 , 3) , (3,1) , (0 , 2) , (2 , 0) ,(2,3)} reflexive , not symmetric , not transitive

Def: The relation R is called equivalence relation if it is reflexive , symmetric , transitive

Remark: Let R_1 , R_2 be two equivalence relations on set A, then

1) $R_1 \cap R_2$ is an equivalence relation on A

2) $R_1 \cup R_2$ is not equivalence relation on A

Chapter four-Function

Def: A relation f from set A to set B is called function if it satisfies

1) ∀ a ∈ A, ∃ b ∈ B such that (a,b) ∈ f

2) If (a,b) ∈ f , (a,c) ∈ f then a = c

A is called domain of f

B is called codomain of f

Remark: f(A) = {b ∈B, (a,b) ∈f} is called range of f

f(A) ⊆B

Example: Let A= {1, 2,3,4} , B ={u,v,w}

f = {(1, u), (2,v),(3,v),(4,w)} is function

Dom f= A , Range f = B

Example: Let A= {1, 2,3,4}

f = {(1,2), (2,3), (3,4),(4,1)} is function

Dom f = A , Range f = B

f_1= {(1,3), (2,4), (3,1)} is not function

f_2 = {(1,4), (2,3), (3,3),(1,2),(4,2)} is not function

Example: Let A = {1, -1,i,-i}, f(x) = x^2

f ={(1,1),(-1,1),(I, -1),(-I,1)}

Domain f = A , Range f = {1,-1}

Every function is relation , the converse is not true

Example: Let A= {a,b,c}, B = {2,3,4, 5}

f = {(a,2), (a,3), (b,4),(c,5)} is relation which is not function

Def: Let A, B be two sets and f : A→B be a function.

1) f is called injective (one to one) if $f(x_1) = f(x_2)$ then $x_1 = x_2$

2) f is called surjective (one to) if for each b ∈ B, there exists a ∈ A such that f(a) = b [f(A)= B]

3| f is called bijective if it is injective and surjective.

Example: f : R→R , f(x) = x^3 is injective and surjective

Example: f : R→R , f(x) = x^2 is not injective and not surjective

Example: Let A= {3, 5,7,9,11}

f = {(3, 5), (5,9),(7,3),(9,5), (11,3)} is not injective and not surjective

Def: Let f : A→B, g : C→D be two function then f = g if

1) A = C
2) B = D
3) f(x) = g(x) ∀x

Example: f : R→R, g : R→R, f(x) = |x| , g(x) = $\sqrt{x^2}$, f = g

Def: Let f : A→B, g : C→D be two functions

Define f× g: A×C→B×D by f×g(a,c) = (f(a), g(c))

1) If f and g are injective then f× g injective

2) If f and g are surjective then f× g surjective

3) If f and g are bijective then f× g bijective

The converse is not true

Def: Let A, B and C be sets and f : A→B , g : B→ C be two functions. The composition function defined as follows: (g∘f)(x) = g(f(x)) ∀x∈ A

Example: Let A = {a,b,c}, B ={x,y,z}, C = {r,s,t,u}

f = {(a,y),(b,x),(c,y)], g = {(x,t),(y,s),(z,u)}

$(g\circ f)(a) = g(f(a)) = g(y) = t$

$(g\circ f)(b) = g(f(b)) = g(z) = s$

$(g\circ f)(c) = g(f(c)) = g(y) = t$

Domain $g\circ f$ = A , Range $g\circ f$ = {s,t}

$g\circ f$ is not injective and not surjective.

Theorem Let A, B and C be sets and $f : A\to B$, $g : B\to C$ be two functions.

1) $(f\circ g)\circ h = f\circ (g\circ h)$

If f and g are injective then $g\circ f$ injective

2) If f and g are surjective then $g\circ f$ surjective

3) If f and g are bijective then $g\circ f$ bijective

4) If $g\circ f$ injective then f injective

5) If $g\circ f$ surjective then g surjective

6) If $g\circ f$ bijective then f injective, g surjective

The converse is not true

Example: $f : R\to R$, $g : C\to R$, $f(x) = 2x^2$, $g(x) = 6x+2$

$(f\circ g)(x) = f(g(x)) = f(6x+2) = 2(6x+2)^2 = 72x^2+48x+8$

$(g\circ f)(x) = g(f(x)) = g(2x^2) = 6(2x^2) + 2 = 12x^2+2$

$g\circ f \neq f\circ g$

Example: Let A= {1, 2,3,4,5}

$f = \{(1, 2), (2,3),(3,4),(4,5), (5,1)\}$

$g = \{(1, 3), (2,4),(3,5),(4,1), (5,2)\}$

$g \circ f = \{(1, 4), (2,5),(3,1),(4,2), (5,3)\}$

$f \circ g = \{(1, 4), (2,5),(3,1),(4,2), (5,3)\}$

$g \circ f = f \circ g$

Def: Let A be any set, a function $f : A \to A$ defined by $f(x) = x$ is called identity function , denoted by I_A

Theorem Let A, B be two sets, $f : A \to B$ be a function then
$f \circ I_A = f$, $I_B \circ f = f$

If $f : A \to B$ be a bijective function then $f^{-1}: B \to A$ be a function

Example: Let A = {a,b,c}, B ={x,y},

f = {(a,x), (b,x), (c,y)}is not bijective

f^{-1} = {(x,a), (x,b), (y,c)} is not function

Example: Let A = {x,y,z}, B ={a,b,c,d},

f = {(x,a), (y,b), (z,c)}is not bijective

f^{-1} = {(a,x), (b,y), (c,z)} is not function

Example: Let A = {a,b,c}, B ={x,y,z},

f = {(a,y), (b,x), (c,z)}is bijective

f^{-1} = {(y,a), (x,b), (z,c)} is function

Proposition Let $f : X \to Y$ be a function has an inverse then each of f , f^{-1} is bijective

Proposition Let $f : X \to Y$ be a function has an inverse, then it is unique

Proposition Let f : X→Y be a function has an inverse then

1) $f^{-1} \circ f = I_A$,

2) $f \circ f^{-1} = I_B$

Proposition Let A, B and C be sets and f : A→B , g : B→C be two functions have an inverse, then the composition function g∘f : A→C has inverse and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$

Def: Let f : X→Y be a function and A⊆X . The set {f(x) ; x∈ A} is called the direct image of the set A under the function f, denoted by f(A)

Def: Let f: X→Y be a function and B ⊆Y. The set {x ∈ X , f(x) ∈ B} is called the opposite image of the set B under the function f, denoted by $f^{-1}(B)$

Example: X = {c,d,e,w} , Y ={1,2,3,4} , A={c,d,e} , B = {1,2}

f ={(c,1), (d,3),(e,2), (w,3)}

f(A) = {f(c), f(d), f(e)}= {1,3,2}

$f^{-1}(B) = \{c,e\}$

Proposition Let f : X→Y be a function has an inverse, B ⊆Y, then $f^{-1}(B) \subseteq X$

Theorem Let f : X→Y be a function and A,B be any two subsets of X

1) $f(A\cup B) = f(A) \cup f(B)$

2) $f(A\cap B) \subseteq f(A) \cap f(B)$

3) $f(A) - f(B) \subseteq f(A - B)$

Example: Let X = {1, 2,3,4} , Y ={a,b}

f = {(1, b), (2,a),(3,b),(4,a)} is function

$A = \{1,2\}$, $B = \{a, b\}$

$A \cap B = \{2\}$, $f(A \cap B) = \{a\}$

$f(A) \cap f(B) = \{a, b\} \cap \{a, b\} = \{a, b\}$

$A - B = \{1\}$, $f(A - B) = \{b\}$

$f(A) - f(B) = \emptyset$

Theorem Let $f : X \rightarrow Y$ be a injective function and A,B be any two subsets of X then $f(A \cap B) = f(A) \cap f(B)$

Theorem Let $f : X \rightarrow Y$ be a function and A,B be any two subsets of Y

1) $f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$

2) $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$

3) $f^{-1}(A) - f^{-1}(B) = f^{-1}(A - B)$