



COMPUTER GRAPHICS

THIRD CLASS

University of Diyala/ College of education for pure
science / Computer science department

23/02/2018

3. Two-Dimensional Transformation (Part 2-2)

4- Mirroring (Reflection):

Mirroring is a convenient method used for copying an object while preserving its features. The mirror transformation is a special case of a negative non-uniform scaling.

If either the X or Y axis is treated as a mirror, the object has a mirror image or reflection. The reflected point P_{new} is located the same distance from the mirror (the axis) as the original point P.

4-1: Reflection on the X axis:

1	0	0	0
0	-1	0	0
0	0	1	0
0	0	0	1

$$\text{OR } \begin{aligned} X_{new} &= X \\ Y_{new} &= -Y \end{aligned}$$

4-2: Reflection on the Y axis:

-1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

$$\text{OR } \begin{aligned} X_{new} &= -X \\ Y_{new} &= Y \end{aligned}$$

4-3: Reflection on the origin:

-1	0	0	0
0	-1	0	0
0	0	1	0
0	0	0	1

$$\text{OR } X_{new} = -X$$

$$Y_{new} = -Y$$

Example 1: Reflect the point P(3,2): a) X axis b) Y axis c) Origin

Solution : a

Solution : a-

$$\begin{bmatrix} 3 & 2 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -2 & 1 \end{bmatrix}$$

b-

$$\begin{bmatrix} 3 & 2 & 1 \end{bmatrix} * \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

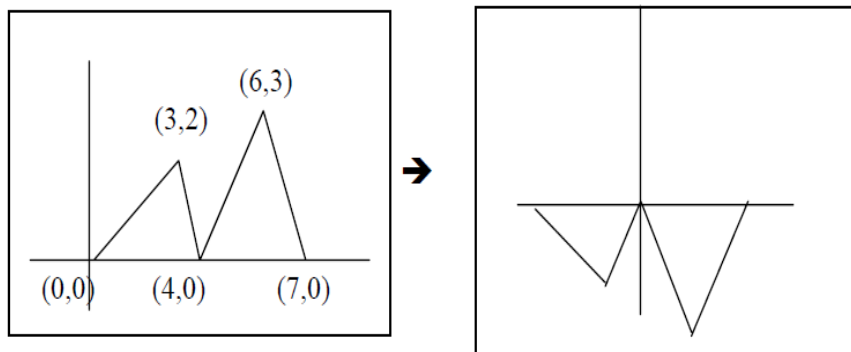
$$= \begin{bmatrix} -3 & 2 & 1 \end{bmatrix}$$

c-

$$\begin{bmatrix} 3 & 2 & 1 \end{bmatrix} * \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

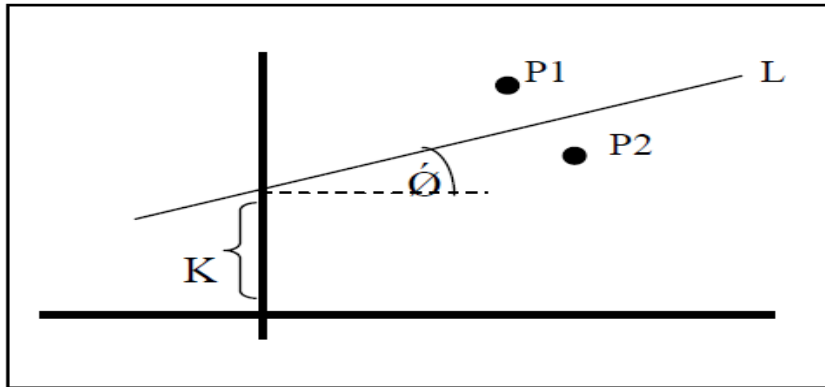
$$= \begin{bmatrix} -3 & -2 & 1 \end{bmatrix}$$

H.W : What (4*4) matrix will change the center of the scene to the origin, and reflect the mountains in the lake? [the center of the scene is (4,0)] .



4-1 Mirroring(reflection) About an Arbitrary line

To reflect an object on a line that does not pass through the origin, which is the general case:



As shown in the figure, let the line L intercept with Y axis in the point (0,K) and have an angle of inclination θ degree with respect to the positive direction of X axis . To reflect the point P1 on the line L, we follow the following steps:

- 1- Move all the points up or down (in the direction of Y axis) so that L pass through the origin

$T=$

1	0	0	0
0	1	0	0
0	0	1	0
0	-K	0	1

- 2- Rotate all the points through $(-\theta)$ degree about the origin making L lie along the X axis

$R=$

$\cos\theta$	$-\sin\theta$	0	0
$\sin\theta$	$\cos\theta$	0	0
0	0	1	0
0	0	0	1

- 3- Reflect the point P1 on the X axis:

$RefX =$

1	0	0	0
0	-1	0	0
0	0	1	0
0	0	0	1

4- Rotate back the points by $(-\theta)$ degree so that L back to its original orientation

 $R^{-1} =$

$\cos\theta$	$\sin\theta$	0	0
$-\sin\theta$	$\cos\theta$	0	0
0	0	1	0
0	0	0	1

5- Shift in the direction of Y axis so that L is back in its original position

 $T^{-1} =$

1	0	0	0
0	1	0	0
0	0	1	0
0	K	0	1

The sequence of matrices needed to perform this non-standard reflection is :

$$S = T * R * RefX * R^{-1} * T^{-1}$$

 $S =$

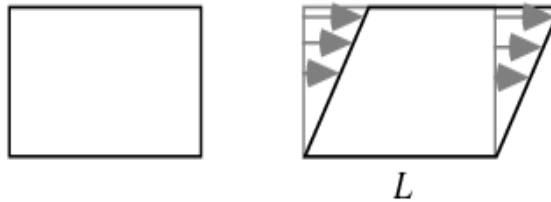
$\cos 2\theta$	$\sin 2\theta$	0	1
$\sin 2\theta$	$-\cos 2\theta$	0	1
$-k\sin 2\theta$	$K + k\cos 2\theta$	1	1
0	0	0	1

Then

$$[P^*] = [P] * S$$

H.W: Find the single matrix that causes all the points in the plane to be reflected about the line with equation $y = 0.5x + 2$, then apply this matrix to reflect the triangle with vertices at A(2,4), B(4,6), C(2,6) about that line.

5-Shearing



A transformation in which all points along a given line L remain fixed while other points are shifted parallel to L by a distance proportional to their perpendicular distance from L . Shearing a plane figure does not change its area.

There are two types of shearing

1- Y shearing

It transforms the point (X, Y) to the point (X_{new}, Y_{new}) where

$$X_{new} = X$$

$$Y_{new} = Y + Sh_y * X \quad \text{where } Sh_y \neq 0$$

The matrix is :

1	Sh_y	0	0
0	1	0	0
0	0	1	0
0	0	0	1

Y shearing moves a vertical line up or down depending on the sign of the shear factor **Sh_y** . A horizontal line is distorted into a line with slope **Sh_y** . And vice versa.

2- X shearing

It transforms the point (X, Y) to the point (X_{new}, Y_{new}) where

$$X_{new} = X + Sh_x * Y \quad \text{where } Sh_x \neq 0$$

$$Y_{new} = Y$$

The matrix is

1	0	0
Shx	1	0
0	0	1

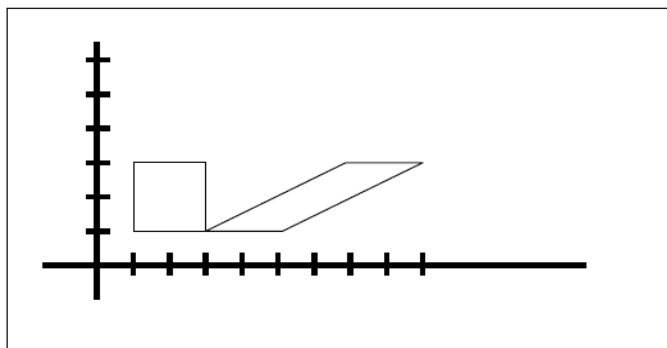
Example : Share the object (1,1) , (3,1) , (1,3) , (3,3) with

a: Shx=2

b: Shy=2

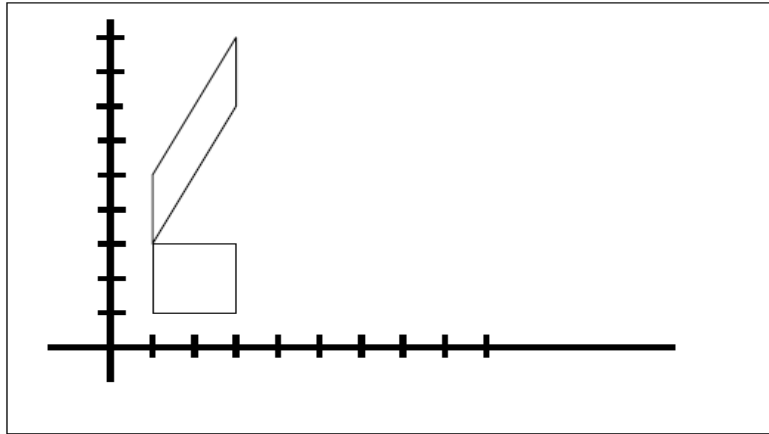
Solution : a: Shx=2

$$\begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 3 & 1 & 1 \\ \hline 1 & 3 & 1 \\ \hline 3 & 3 & 1 \\ \hline \end{array} * \begin{array}{|c|c|c|} \hline 1 & 0 & 0 \\ \hline 2 & 1 & 0 \\ \hline 0 & 0 & 1 \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline 3 & 1 & 1 \\ \hline 5 & 1 & 1 \\ \hline 7 & 3 & 1 \\ \hline 9 & 3 & 1 \\ \hline \end{array}$$



b- Shy

$$\begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 3 & 1 & 1 \\ \hline 1 & 3 & 1 \\ \hline 3 & 3 & 1 \\ \hline \end{array} * \begin{array}{|c|c|c|} \hline 1 & 2 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 1 \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline 1 & 3 & 1 \\ \hline 3 & 7 & 1 \\ \hline 1 & 5 & 1 \\ \hline 3 & 9 & 1 \\ \hline \end{array}$$



Quiz: Generate figure 2 from figure 1 using matrices .

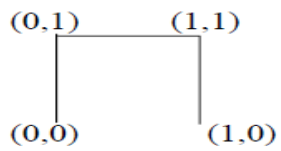


Figure 1

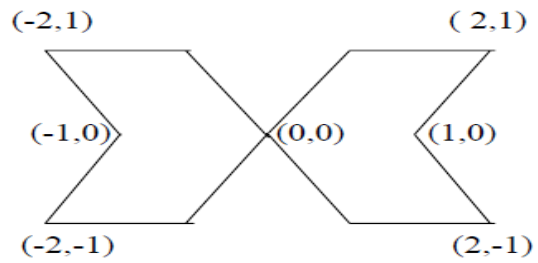


Figure 2