Chapter 2
Functions

1.2 Def:- A function f from a set A to aset B “written $f \cdot D \to R$ “ is a rule which assigns a single element y∈B to each element x∈A .

Notes:

The element y∈B denoted f(x).

The set A is called the domain of f.

The set R is called the range of f.

Ex(3) Given the function f(x)=x^2-2x+3. find f(-1), f(0), f(2), f(x+1), f(f(x)), f(f(1))

Solu**.** f(x)=x^2-2x+3

$f(-1)=(-1)^2-2(-1)+3=1+2+3=6$

$f(0)=0-0+3=3$

$f(2)=4-4+3=3$

$f(x+1)=(x+1)^2-2(x+1)+3=x^2+2x+1-2x-2+3=x^2+2$

$f(f(x))=(f(x))^2-2(f(x))+3=(x^2-2x+3)^2-2(x^2-2x+3)+3$

$=x^4+4x^2+9-4x^3+6x^2-12x-2x^2+4x-6+3=x^4-4x^3+8x^2+8x+6$

$f(f(1))=1^4-4(1)^3+8(1)^2+8(1)+8=1-4+8-8+6=3$

Ex(4) Find the domain and range of the following functions:-

1. y=f(x)=x^2 , D :all x or D : $-\infty<x<\infty$

$x=\pm\sqrt{y}$, R: $y\geq 0$

2. $y=\dfrac{x-1}{x-6}$, D: $x\neq 6$

yx-6y=x-1

yx-x=6y-1

$x=\dfrac{6y-1}{y-1}$, R: $y\neq 1$

3. $y=\sqrt{1-x^2}$

$1-x^2\geq 0 \Rightarrow (1-x)(1+x)\geq 0$

D: $-1\leq x\leq 1$

$y^2=1-x^2 \Rightarrow x^2=1-y^2 \Rightarrow x=\sqrt{1-y^2}$

R: $-1\leq y\leq 1$

4. $y=\sqrt{x-3}$

$x-3\geq 0 \Rightarrow x\geq 3$

D: $(3,\infty)$

R: $(0,\infty)$

5. $y=\sqrt{x^2-4x+3}$

$x^2-4x+3\geq 0 \Rightarrow (x-3)(x-1)\geq 0$

D: $x\leq 1$ or $x\geq 3$

$y^2=x^2-4x+3 \Rightarrow x^2-4x+3-y^2=0$

$$x=\frac{4\pm\sqrt{16-4*1(3-y^2)}}{2*1} \Rightarrow x=\frac{4\pm\sqrt{16-12+4y^2}}{2}$$

$$\Rightarrow x=\frac{4\pm\sqrt{4-4y^2}}{2} \Rightarrow x=\frac{4\pm 2\sqrt{1+y^2}}{2} \Rightarrow x=2\pm\sqrt{1+y^2}$$

R: all y or $-\infty<y<\infty$

6. $y=\sqrt{2-\sqrt{x}}$

$2-\sqrt{x}\geq 0 \Rightarrow 2\geq\sqrt{x} \Rightarrow 4\geq x$

D: $0\leq x\leq 4$

$y^2=2-\sqrt{x} \Rightarrow \sqrt{x}=2-y^2 \Rightarrow x=(2-y^2)^2$

R: all y

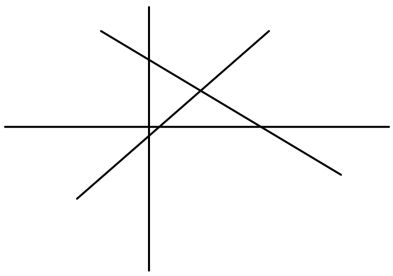
1.3 Graph of function

Def:- The solution set or locus of an equation, In two unknown consists of all point in plane whose coordinates satisfy the equation.

A geometrical representation of the locus is called the graph of the equation.

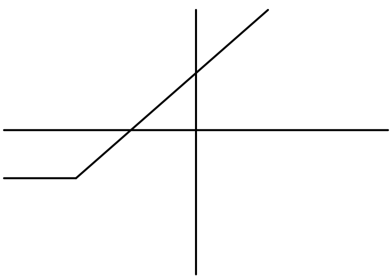
Ex(5) Sketch the graph of the following functions:-

a. $y=\begin{cases} x, 0\leq x<1 \\ 2-x, 1<x\leq 2 \end{cases}$



Ex(6) sketch the graph of the function $y=|x+2|+x$ for $-5\leq x\leq 2$

solu.



$$y = |x+2| + x = \begin{cases} (x+2)+x, x+2 \geq 0 \\ -(x+2)+x, x+2 < 0 \end{cases}$$

$$y = \begin{cases} 2x+2, x \geq -2 \\ -2, x < -2 \end{cases}$$

1.6 Intercepts, Symmetry and more about graphing

1. To find x-intercepts, set y=0 and solve for x.
To find y-intercepts, set x=0 and solve for y.

2. The locus is symmetric w.r.t the:-
a. x-axis $\Leftrightarrow$ (x,y)≡(x,-y)
b. y-axis $\Leftrightarrow$ (x,y)≡(-x,y)
c. origin $\Leftrightarrow$ (x,y)≡(-x,-y)

Ex(7) Find the domain ,range, intercepts, symmetry, for the following functions.

1. $y=f(x)=x^3-x$
D : all x
R : all y

Intercepts

Set y=0 $\Rightarrow$ $x^3-x=0$ $\Rightarrow$ $x(x^2-1)=0$ $\Rightarrow$ x(x-1)(x+1)=0 $\Rightarrow$ x=0,1,-1
x-int. are: (0,0),(1,0),(-1,0)
Set x=0 $\Rightarrow$ y=0
y-int. is:(0,0)

Symm.

$y=x^3-x$
about x-axis: $-y=x^3-x$ $\Rightarrow$ $y=-x^3+x$
No symm. w.r.t x-axis
about y-axis: $y=(-x)^3-(-x)=-x^3+x$
about origin:

2. $f(x) = \dfrac{1}{x^2-1}$

D: $x \neq \pm 1$

$yx^2-y=1$ $\Rightarrow$ $yx^2=y+1$ $\Rightarrow$ $x^2 = \dfrac{y+1}{y}$

$x = \pm\sqrt{\dfrac{y+1}{y}} \Rightarrow \dfrac{y+1}{y} \geq 0$

R: $y \leq -1$ or y>0

Note :- symmetry and intercepts Can provide additional guidance in graphing.

1.5 Algebra of functions (sums, Differences, products, Quotients)

Let f(x) and g(x) be functions then:-

1. f+g is a function such that (f+g)(x)=f(x)+g(x)
2. f-g is a function such that (f-g)(x)=f(x)-g(x)
3. f.g is a function such that (f.g)(x)=f(x).g(x)
4. f/g is a function such that (f/g)(x)=f(x)/g(x), $g(x) \neq 0$

Notes: 1. $D_{f+g} = D_{f-g} = D_{f.g} = D_f \cap D_g$

2. $D_{f/g} = (D_f \cap D_g)/\{x \in D_g / g(x) = 0\}$

Ex(8) let $f(x) = \dfrac{x}{x+1}$ and $g(x) = \dfrac{x-1}{x}$ be functions find the domain of f(x), g(x) and D_{f+g}, D_{f-g}, $D_{f.g}$, $D_{f/g}$

Solu. $D_f = R/\{-1\}$, $D_g = R/\{0\}$

$D_f \cap D_g = R/\{0,1\}$

$$(f+g)(x) = \frac{x}{x+1} + \frac{x-1}{x} = \frac{x^2 + x^2 - 1}{x(x+1)} = \frac{2x^2 - 1}{x(x+1)}$$

$$(f-g)(x) = f(x) - g(x) = \frac{x}{x+1} - \frac{x-1}{x} = \frac{x^2 - x^2 + 1}{x(x+1)}$$

$$(f.g)(x) = f(x).g(x) = \frac{x}{x+1} . \frac{x-1}{x} = \frac{x-1}{x+1}, \quad x \neq -1$$

$$D_{f+g} = D_{f-g} = D_{f.g} = R/\{0,-1\}$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\frac{x}{x+1}}{\frac{x-1}{x}} = \frac{x^2}{x^2-1}$$

$$D_{f/g} = D_f \cap D_g / \{x; g(x) = 0\}$$

$$= R/\{0,-1\}/\{x; x^2 - 1 = 0\} = R/\{0,-1,1\}$$

Def **1-** f(x) is an odd function if f(-x)=-f(x) $\Rightarrow$ f(x) is symmetric about the origin

2- f(x) is an even function if f(-x)=f(x) $\Rightarrow$ f(x) is symmetric about the y-axis

Ex(9)

1. $f(x)=\dfrac{x^2-1}{\sin x}$

$$f(-x)=\frac{(-x)^2-1}{\sin(-x)}=\frac{x^2-1}{-\sin x}=\frac{x^2-1}{\sin x}=-f(x)$$

$\therefore$ f(x) is odd function

2. f(x)=x^2cosx

f(-x)=$(-x)^2$cos(-x)

=x^2cosx=f(x)

$\therefore$ f(x) is even function

Ex(10) $f(x)=x^2+2$

$f(-x)=(-x)^2+2=x^2+2=f(x)$

f(x) is even function

Ex(11) $f(x)=x^3$

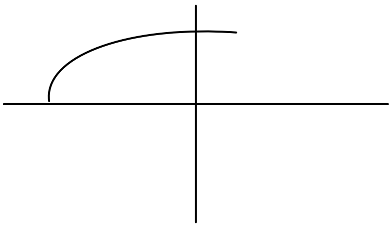
$f(-x)=(-x)^3=-x^3=-f(x)$

f(x) is odd function

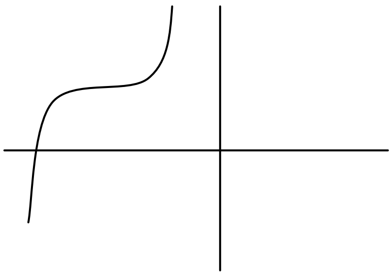
1.6 Graphs and Graphing

The points in the plane whose coordinates (x,y) are the input – output pairs of a function make up the graph of the function, Thus the graph of a function defined by an equation in x and y is the graph of the equation itself. We make a table of matching xy- pairs and connect the points (x,y) with a smooth curve.

Ex(12) sketch the graph of $y=\sqrt{x+3}$



Ex(13) sketch the graph of $y=\sqrt[3]{4+x}+2$



The greatest integer function (step function)

The greatest integer less than or equal to a number x is called the greatest integer in x. because each real number x corresponds to only one greatest integer, the greatest integer in x is a function of x. The symbol for it is [x] which read the greatest integer in x $\Rightarrow [x] \le x,\ f : x \to [x]$

some values of y=[x]

Positive [1.8]=1
[2.3]=2
Zero [0.5]=0
[0]=0
Negative [-1.2]=-2
[-0.5]=-1

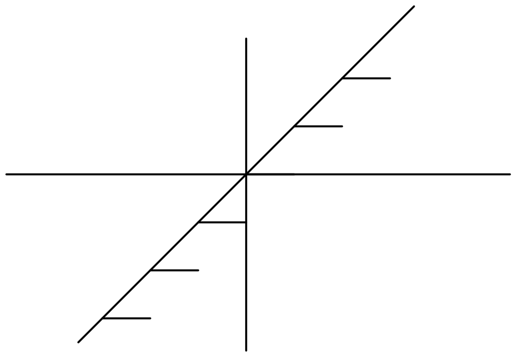
Note: y=f(x)=[x]

$D_f = (-\infty, \infty)$

$R_f = \{y, y \in z\}$

Ex(14) sketch the graph of y=[x], $-3 \le x \le 3$

X	y=[x]	(x,[x])
$-3 \le x \le -2$	-3	(-3,-3)(-2,-3)
$-2 \le x \le -1$	-2	(-2,-2)(-1,-2)
$-1 \le x \le 0$	-1	(-1,-1)(0,-1)
$0 \le x \le 1$	0	(0,0)(1,0)
$1 \le x \le 2$	1	(1,1)(2,1)
$2 \le x \le 3$	2	(2,2)(3,2)



1.7

Exercises

1. Find the domain and range of each function

1. $y = \sqrt{x+4}$ **2.** $y = \sqrt{-x}$ **3.** $y = \dfrac{1}{1-x^2}$

4. $y = \sqrt{x - x^2}$ **6.** $y = \dfrac{\sqrt{x-1}}{x-2}$

5. $y = \dfrac{x^2}{1+x^2}$

7. **8.** $y = x - \dfrac{1}{x}$

2.

3. Graph the following functions

1. $y = 4 - x^2$ **2.** $y = -\dfrac{1}{x}$ **3.** $y = |x-2|$

4. $y=[x+3]-1, -2\le x\le 2$

5. $y=\dfrac{[x]}{x}, x\ne 0, -3\le x\le 3$

6. $y=[x]+[-x], -3\le x\le 3$

7. $y=x-[x], -3\le x\le 3$

Limits and Continuity

Def.:- if the values of a function f of x approach the value L as x approaches a, we say F has limit L as x approaches a and we write $\lim\limits_{x\to a} f(x)=L$.

Ex(1) Let f(x)=2x+5 . evaluate f(x) at x=1.1,1.01,1.001,…

f(1.1)=2(1.1)+5=7.2
f(1.01)=2(1.01)+5=7.02
f(1.001)=2(1.001)+5=7.002
⋮

we see that f(x) tends to 7 as x approach to 1, so we say $f(x)\to 7$ as $x\to 1$

Theorems on Limits

1. Ungeness of limit

If $\lim\limits_{x\to a} f(x)=L_1$ and $\lim\limits_{x\to a} f(x)=L_2$, then $L_1=L_2$

2. Limit of constant

If f(x)=c where c is constant ,then $\lim\limits_{x\to a} f(x)=\lim\limits_{x\to a} c=c$

3. Obvious limit

If f(x)=x then $\lim\limits_{x\to a} f(x)=\lim\limits_{x\to a} x=a$

4. limit of a sums and differences

If $f(x)=f_1(x)\mp f_2(x)\mp\ldots\mp f_n(x)$ and $\lim\limits_{x\to a} f_i(x)=L_i$,i=(1,2,…,n)

$$\lim_{x\to a} f(x)=\lim_{x\to a}[f_1(x)\mp f_2(x)\mp\ldots\mp f_n(x)]=\lim_{x\to a} f_1(x)\mp\lim_{x\to a} f_2(x)\mp\ldots\mp\lim_{x\to a} f_n(x)$$

$$=L_1\mp L_2\mp\ldots\mp L_n=\sum_{i=1}^{n} L_i$$

5. Limit of a products

If $f(x)=f_1(x)\times f_2(x)\times\ldots\times f_n(x)$ and $\lim f_i(x)=L_i$, i=(1,2,…, n)

then

$$\lim_{x\to a} f(x)=\lim_{x\to a}[f_1(x)\cdot f_2(x)\cdot\ldots\cdot f_n(x)]=\lim_{x\to a} f_1(x)\cdot\lim_{x\to a} f_2(x)\cdot\ldots\cdot\lim_{x\to a} f_n(x)$$

$$=L_1\cdot L_2\cdot\ldots\cdot L_n=\prod_{i=1}^{n} L_i$$

6. Limit of quotients

If $f(x)=\dfrac{g(x)}{h(x)}$ and $\lim\limits_{x\to a} g(x)=L_1$ and $\lim\limits_{x\to a} h(x)=L_2$

then $\lim_{x\to a} f(x) = \lim_{x\to a} \frac{g(x)}{h(x)} = \frac{\lim_{x\to a} g(x)}{\lim_{x\to a} h(x)} = \frac{L_1}{L_2}, \quad h(x) \neq 0$

the limits are all taken as $\mathbf{x} \to a$ and L_1 and L_2 are real numbers.

Ex(2) Evaluate the following limits.

1. $\lim_{x\to -1} 3x^2 = 3\lim_{x\to -1} x^2 = 3(-1)^2 = 3$

2. $\lim_{x\to 1}(4x^2+5) = \lim_{x\to 1} 4x^2 + \lim_{x\to 1} 5 = 4+5 = 9$

$$\lim_{x\to 2}\frac{x^3+4x^2-3}{x^2+5} = \frac{\lim_{x\to 2} x^3+4x^2-3}{\lim_{x\to 2} x^2+5} = \frac{(2)^3+4(2)^2-3}{(2)^2+5} = \frac{8+16-3}{9} = \frac{21}{9} = \frac{7}{3}$$

Ex(3) Evaluate the following limits if they exist :-

a. $\lim_{x\to 2}\frac{x^2-3x+2}{x-2}$

$D = R \neq 2$, so the denominator is 0 when x=2

$$\lim_{x\to 2}\frac{x^2-3x+2}{x-2} = \frac{4-3(2)+2}{2-2} = \frac{0}{0}$$

We have to rewrite the fraction $\frac{x^2-3x+2}{x-2}$ first ,then

$$\lim_{x\to 2} f(x) = \lim_{x\to 2}\frac{x^2-3x+2}{x-2} = \lim_{x\to 2}\frac{(x-2)(x-1)}{(x-2)} = \lim_{x\to 2}(x-1) = (2-1) = 1$$

b. $\lim_{x\to 2}\frac{x^3-8}{x^2-4} = \frac{(2)^3-8}{(2)^2-4} = \frac{0}{0}$

$$\lim_{x\to 2}\frac{x^3-8}{x^2-4} = \lim_{x\to 2}\frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)} = \lim_{x\to 2}\frac{x^2+2x+4}{x+2} = \frac{(2)^2+2(2)+4}{(2)+2} = \frac{12}{4} = 3$$

Ex(4) Evaluate the following limits if they exist :-

a. $\lim_{x\to -1}\frac{\sqrt{2+x}-1}{x+1}, x \neq -1, x \geq -2$

$$\lim_{x\to -1}\frac{\sqrt{2+x}-1}{x+1}\cdot\frac{\sqrt{2+x}+1}{\sqrt{2+x}+1} = \lim_{x\to -1}\frac{(2+x-1)}{(x+1)(\sqrt{2+x}+1)} = \lim_{x\to -1}\frac{1}{(\sqrt{2+x}+1)} = \frac{1}{\sqrt{2-1}+1}$$

$$= \frac{1}{2}$$

b. $\lim_{x\to 2}\frac{2-x}{2-\sqrt{2x}}, x \neq 2, x \geq 0$

$$\lim_{x\to 2}\frac{2-x}{2-\sqrt{2x}}\cdot\frac{2+\sqrt{2x}}{2+\sqrt{2x}} = \lim_{x\to 2}\frac{(2-x)(2+\sqrt{2x})}{4-2x} = \lim_{x\to 2}\frac{2+\sqrt{2x}}{2} = \frac{2+\sqrt{4}}{2} = \frac{4}{2} = 2$$

Ex(5) Evaluate the following limits

a. $\lim_{x\to 1}\frac{x^3-1}{x-1}, x \neq 1$

$$=\lim_{x\to 1}\frac{(x-1)(x^2+x-1)}{(x-1)}=\lim_{x\to 1}(x^2+x-1)=3$$

b. $\lim_{h\to 0}\frac{1}{h}(\frac{1}{x+h}-\frac{1}{x}), h\neq 0$

$$=\lim_{h\to 0}\frac{1}{h}(\frac{x-x-h}{(x+h)x})=-\lim_{x\to 0}\frac{1}{x(x+h)}=-\frac{1}{x(x+0)}=-\frac{1}{x^2}$$

One-sided and two-sided limits(Right and Left limits)

Some times the values of a function f(x) tends to different limits as x approaches a from different sides, when this happens, we call the limit of f(x) as x approach a from the right (the right hand limit of f at a) and denoted by $\lim_{x\to a^+} f(x)$ and (+) means that x approaches a through values above a on the number line.

And the limit of f(x) as x approaches a from the left (left hand limit of f at a) and denoted by $\lim_{x\to a^-} f(x)$ and (-) means that x approaches a through values below a on the number line.

Note**:** A function f has a limit as x approaches a if and only if the right-hand and left-hand limits at a exist and are equal in symbols. $\lim_{x\to a} f(x)=L \Leftrightarrow \lim_{x\to a^+} f(x)=L$ and $\lim_{x\to a^-} f(x)=L$.

Ex(6) let $f(x)=\sqrt{x}$ find $\lim_{x\to 0} f(x)=2$

$$\lim_{x\to 0} f(x)=\lim_{x\to 0}\sqrt{x}=0$$

we shall explain this limit

$f(x)=\sqrt{x} \quad D: x\geq 0$

since $\sqrt{x}$ is not defined for –ive values

so we restrict to +ive values of x

$$\lim_{x\to 0^+} f(x)=\lim_{x\to 0}\sqrt{x}=0=\lim_{x\to 0} f(x)$$

Note: this example of one-sided limits

Ex(7) If $f(x)=\sqrt{1-x}$ find $\lim_{x\to 1} f(x)$

$$\lim_{x\to 1} f(x)=\lim_{x\to 0}\sqrt{1-x}=\sqrt{1-1}=0$$

we shall explain this example

$f(x)=\sqrt{1-x}$, $D: 1-x\geq 0 \Rightarrow x\leq 1$

since $\sqrt{1-x}$ is not defined for x>1

so we restrict on values of $x\leq 1$

$$\lim_{x\to 1^-} f(x)=\lim_{x\to 1^-}\sqrt{1-x}=\sqrt{1-1}=0=\lim_{x\to 1} f(x)$$

this example of one-sided limits

Ex(8) let $f(x)=\frac{x}{|x|}$ find $\lim_{x\to 0} f(x)$

$$f(x)=\frac{x}{|x|}=\begin{cases}\frac{x}{x}, x\geq 0\\ -\frac{x}{x}, x<0\end{cases}=\begin{cases}1, & x>0\\ -1, & x<0\end{cases}$$

$\lim_{x\to 0^+} f(x)=\lim_{x\to 0^+}(1)=1$, $\lim_{x\to 0^-} f(x)=\lim_{x\to 0^-}(-1)=-1$

$\lim_{x\to 0^+} f(x)\neq \lim_{x\to 0^-} f(x)$

$\therefore \lim_{x\to 0} f(x)=\lim_{x\to 0}\frac{x}{|x|}$ does not exist .

this example of two-sided limits

Exercises

a. $\lim_{x\to 2}\frac{x^4-2x^2-8}{x^2-4}, x\neq 2$

b. $\lim_{x\to a}\frac{\sqrt{x^2+1}-\sqrt{a^2+1}}{x-a}, x\neq a$

c. $\lim_{x\to 3}\frac{\sqrt{3x}-3}{x-3}, x\neq 3$

d. $\lim_{x\to 0}\frac{1}{x}(\frac{1}{x+2}-\frac{1}{2})$

e. $\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}, h\neq 0$

Limits of rational functions as $x\to\mp\infty$

To find the limit of a rational function as $x\to\mp\infty$(when the limit exists) we divide the numerator and denominator by the highest power of x.

Ex(10) Find the following limits :-

1. $\lim_{x\to\infty}\frac{x}{2x+3}=\lim_{x\to\infty}\frac{\frac{x}{x}}{\frac{2x+3}{x}}=\lim_{x\to\infty}\frac{1}{2+\frac{3}{x}}=\frac{1}{2+\frac{3}{\infty}}=\frac{1}{2+0}=\frac{1}{2}$

2. $\lim_{x\to\infty}\frac{2x^2+1}{3x^3-2x^2+5x-2}=\lim_{x\to\infty}\frac{\frac{2}{x}-\frac{1}{x^2}}{3-\frac{2}{x}+\frac{5}{x^2}-\frac{2}{x^3}}=\frac{0+0}{3-0+0-0}=\frac{0}{3}=0$

3. $\lim_{x\to\infty} \frac{2x^3+3x^2+1}{x^2-5x+2} = \lim_{x\to\infty} \frac{2+\frac{3}{x}+\frac{1}{x^3}}{\frac{1}{x}-\frac{5}{x^2}+\frac{2}{x^3}} = \frac{2+0+0}{0-0+0} = \frac{2}{0} = \infty$

4. $\lim_{x\to-\infty} \frac{2x^2-3}{7x+4} = \lim_{x\to-\infty} \frac{2x-\frac{3}{x}}{7+\frac{4}{x}} = -\infty$

5. $\lim_{x\to\infty} \sqrt{x} = \lim_{x\to+\infty} \sqrt{x} = +\infty$

6. $\lim_{x\to\infty} (2+\frac{\sin x}{x}) = \lim_{x\to\infty} (2) + \lim_{x\to\infty} \frac{\sin x}{x}$

but $\lim_{x\to\infty} (2) = 2$ and $\lim_{x\to\infty} \frac{\sin x}{x} = 0$ because $-1 \le \sin x \le 1$

$\lim_{x\to\infty} (2+\frac{\sin x}{x}) = 2+0 = 2$

7. $\lim_{x\to-\infty} (2x+\frac{3}{x}) = \lim_{x\to-\infty} 2x + \lim_{x\to-\infty} \frac{3}{x} = -\infty + 0 = -\infty$

8. $\lim_{x\to 2^-} \frac{1}{x^2-4} = \frac{1}{4-4} = \frac{1}{0} = -\infty$

Continuity

Def (Continuous function)

A function f(x) is continuous at x=a if and only if the following statements are true.

1. f(a) exists (a lies in the domain of f).

2. $\lim_{x\to a} f(x)$ exists (f has a limit as $x \to a$).

3. $\lim_{x\to a} f(x) = f(a)$ (the limit equals the function value).

Ex(13)

1. Every polynomial of the from

$f(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$ is cont. for all x

2. The function $y = \frac{1}{x}$ is continuous at every value of x except x=0 because

$f(0) = \frac{1}{0} = \infty$ so the function is not defined at x=0.

3. $f(x) = \frac{x+3}{(x-1)(x+2)}$ is continuous at every value of x except x=1 and x=-2.

4. let $f(x)=\begin{cases}\dfrac{x^2+x-6}{x^2-4}, x\neq 2\\ \dfrac{5}{4}, x=2\end{cases}$ is f(x) cont. at x=2 ?

1- $f(2)=\dfrac{5}{4}$

2- $\lim\limits_{x\to 2} f(x)=\lim\limits_{x\to 2}\dfrac{x^2+x-6}{x^2-4}=\lim\limits_{x\to 2}\dfrac{(x-2)(x+3)}{(x-2)(x+2)}=\lim\limits_{x\to 2}\dfrac{x+3}{x+2}=\dfrac{2+3}{2+2}=\dfrac{5}{4}=f(2)$

$\therefore$ f(x) is cont. of x=2

5. $f(x)=\dfrac{\sin x}{x}$ is continuous at every value of x except x=0.

6. $f(x)=\begin{cases}\dfrac{\sin x}{x}, x\neq 0\\ 1 \quad ,x=0\end{cases}$

1- $f(0)=1$

2- $\lim f(x)=\lim\dfrac{\sin x}{x}=1=f(0)$

$\therefore f(x)$ is continuous at x=0

Exercises

1. What value should be assigned to a to make the function

$f(x)=\begin{cases}x^2-1, & x<3\\ 2ax, & x\geq 3\end{cases}$ continuous at x=3 ?

2. Is $f(x)=\begin{cases}x^2+2, & x\geq 2\\ 2x+2, & x<2\end{cases}$ continuous at x=2 ?

3. Find the points, at which the following functions are not continuous?

a. $y=\dfrac{1}{x-2}$

b. $y=\dfrac{1}{(x+2)^2}$

c. $y=|x-1|$

d. $y=\dfrac{x+1}{x^2-4x+3}$

e. $y=\dfrac{1}{x^2+1}$