

CHAPTER ONE

Numbers

1.1 Inequalities

If a and b are real numbers then one of the following is true. $a > b$ or $a = b$ or $a < b$

Notes:

1. If $a > b$ then $-a < -b$
 $4 > 2$ then $-4 < -2$
2. If $a > b$ then $1/a < 1/b$
 $4 > 2$ then $1/4 < 1/2$

Intervals

An interval is a set of numbers x having one of the following forms:-

1. Open interval: $a < x < b \equiv (a, b)$
2. Close interval: $a \leq x \leq b \equiv [a, b]$
3. Half open from the left or half close from the right: $a < x \leq b \equiv (a, b]$
4. Half close from the left or half open from the right: $a \leq x < b \equiv [a, b)$

Notes:

1. $a < x < \infty \equiv a < x \equiv (a, \infty)$
2. $a \leq x < \infty \equiv a \leq x \equiv [a, \infty]$
3. $-\infty < x < a \equiv x < a \equiv (-\infty, a)$
4. $-\infty < x \leq a \equiv x \leq a$
5. $-\infty < x < \infty \equiv (-\infty, \infty)$

Ex(1) Find the solution set of the following inequalities:-

1. $12x - 21 < 27 + 4x$
 $12x - 4x < 27 + 21$
 $8x < 48$
 $x < 48/8 = 6$
 solution set is $-\infty < x < 6$
2. $x^2 - x - 12 < 0$
 $(x+3)(x-4) < 0$
 solution set is $-3 < x < 4$
3. $2x^2 + 5x + 2 > 0$
 $(2x+1)(x+2) > 0$
 solution set is $x < -2$ or $x > -1/2$
4. $x^3 - x \leq 0$
 $x(x^2 - 1) \leq 0$
 $x(x-1)(x+1) \leq 0$
 solution set is $x \leq -1$ or $0 \leq x \leq 1$

$$5. x^2 + 2x + 2 > 0$$

$$x^2 + 2x + 1 + 1 > 0$$

$$(x+1)^2 + 1 > 0$$

solution set is $-\infty < x < \infty$

$$6. \frac{x-1}{x^2+x-6} < 0$$

$$\frac{x-1}{(x+3)(x-2)} < 0$$

solution set is: $x < -3$ or $1 < x < 2$

Absolute Value

The absolute value of the real number x is defined as:-

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$|3| = 3$$

$$|-3| = -|-3| = 3$$

In general

$$|u(x)| = \begin{cases} u(x), & u(x) \geq 0 \\ -u(x), & u(x) < 0 \end{cases}$$

Properties of the absolute value

$$1. |x \cdot y| = |x| \cdot |y|$$

$$2. |x| = |-x|$$

$$3. |x + y| \leq |x| + |y|$$

$$4. |x| < a \text{ mean } -a < x < a$$

$$5. |x| \leq a \text{ mean } -a \leq x \leq a$$

$$6. |x| > a \text{ mean } x < -a \text{ or } x > a$$

$$7. |x| \geq a \text{ mean } x \leq -a \text{ or } x \geq a$$

Ex(2) Find the solution set of the following inequalities:-

$$a. \left| \frac{3x+1}{2} \right| < 1$$

solu.

$$-1 < \frac{3x+1}{2} < 1 \Rightarrow -2 < 3x+1 < 2 \Rightarrow -2-1 < 3x < 2-1 \Rightarrow -3 < 3x < 1$$

$$\Rightarrow -1 < x < 1/3$$

$$b. |x-1| \geq 5$$

solu.

$$|x-1| \geq 5 \Rightarrow x-1 \leq -5 \text{ or } |x-1| \geq 5 \Rightarrow x-1 \geq -5 \Rightarrow x \leq -4 \text{ or } x \geq 6$$

